MULTIBARYONS WITH STRANGENESS, CHARM AND BOTTOM

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Static properties of multiskyrmions with baryon numbers up to 8 are calculated, including momenta of inertia and sigma-term. The calculations are based on the recently suggested $SU(2)$ rational map ansaetze. The spectra of baryonic systems with strangeness, charm and bottom are considered within a "rigid oscillator" version of the bound state soliton model. The binding energies estimates are made for the states with largest isospin which can appear as negatively charged nuclear fragments, as well as for states with zero isospin - light fragments of "flavoured" nuclear matter. Our results confirm the previously made observation that baryonic systems with charm or bottom quantum numbers have more chance to be stable with respect to strong interactions than strange baryonic systems.

1 Introduction

The topological soliton models, and the Skyrme model among them [1], are attractive because of their simplicity and the possibility that they may describe well various properties of low energy baryons. The models of this kind provide also a very good framework within which to investigate the possibility of the existence of nuclear matter fragments with unusual properties, such as flavour being different from u and d quarks. In addition to being important by itself, this issue can have important consequences in astrophysics and cosmology. In particular, the formation and subsequent decay of such fragments could be important in the early stages of the evolution of the Universe. It is well known that the relativistic many-body problems cannot be solved directly using the existing methods, and the chiral soliton approach may allow us to overcome some of these difficulties.

The description of skyrmions with large baryon numbers has been perceived as being complicated because the explicit form of the fields has been not known. A recent observation [2] that the fields of the $SU(2)$ skyrmions can be approximated accurately by rational map ansaetze giving the values of masses close to their precise values, has simplified considerably the task of such studies. Similar ansaetze have also been recently presented for $SU(N)$ skyrmions (which are not embeddings of $SU(2)$ fields)[3].

In this paper we use the $SU(2)$ rational map ansaetze as the starting points for the calculation of static properties of bound states of skyrmions necessary for their quantization in the $SU(3)$ collective coordinate space. The energy density of the $B = 3$ configuration has tetrahedral symmetry, of $B = 4$ - the octahedral (cubic) one [4], of $B = 5$ - D_{2d}-symmetry, of $B = 6$ - D_{4d}, of $B = 7$ - dodecahedral symmetry, and of $B = 8$ - D_{6d} - symmetry [5, 2], etc.

The minimization, with the help of a 3-dimensional variational program [6], lowers the energies of these configurations by few hundreds of Mev and shows that they become local minima in the $SU(3)$ configuration space. The knowledge of the so-called "flavour" moment of inertia and the Σ-term allows us then to estimate the flavour excitation energies. The mass splittings of the lowest states with different values of strangeness, charm or bottom are calculated within the rigid oscillator version of the bound state approach. The binding energies of baryonic systems (BS) with different values of flavours are also estimated.

To reduce theoretical uncertainties we consider the differencies between the binding

energies of BS with flavour F and the ground state for each value of B. These ground states are the deuteron for $B = 2$, the isodoublet ^3H-^3He for $B = 3$, ^4He for $B = 4$, etc. These differencies, being free of many uncertainties, in particular of the poorly known loop corrections to classical masses, show the tendency of the flavoured BS to be more bound than the (u, d) ground states (for heavy flavours), or to be less bound, as for strangeness. Of course, it is an assumption that the ground states of multiskyrmions correspond to ordinary nuclei. However, this is a natural assumption if we believe that effective field theories describe nature.

In the next Section the static characteristics of multiskyrmions are described. Flavour excitation energies and zero mode corrections to the energies of multibaryons are considered in Section 3. Section 4 contains estimates for the binding energies of baryonic systems with different values of flavours, and our conclusions are given in Section 5.

2 Static characteristics of multiskyrmions

We consider here simple $SU(3)$ extensions of the Skyrme model [1]: we start with $SU(2)$ skyrmions (with flavour corresponding to (u, d) quarks) and extend them to various $SU(3)$ groups, such as, (u, d, s), (u, d, c), or (u, d, b).

We take the Lagrangian of the Skyrme model which, in its well known form, depends on parameters F_π, F_D and e and can be written in the following way [7]:

$$\mathcal{L} = \frac{F_\pi^2}{16} Tr l_\mu l^\mu + \frac{1}{32e^2} Tr[l_\mu, l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{16} Tr(U + U^\dagger - 2) +$$

$$+ \frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24} Tr(1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2) + \frac{F_D^2 - F_\pi^2}{48} Tr(1 - \sqrt{3}\lambda_8)(U l_\mu l^\mu + l_\mu l^\mu U^\dagger). \quad (1)$$

Here $U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields, and $l_\mu = U^\dagger \partial_\mu U$. In this model F_π is fixed at the physical value: $F_\pi = 186$ Mev and M_D is the mass of K, D or B meson. The ratios F_D/F_π are known to be 1.22 and 1.7 ± 0.2 for, respectively, kaons and D-mesons.

The flavour symmetry breaking (FSB) in the Lagrangian is of the usual form, and is sufficient to describe the mass splittings of the octet and decuplets of baryons within the collective coordinate quantization approach [7].

The Wess-Zumino term, to be added to the action, which can be written as a 5-dimensional differential form [8] plays an important role in the quantization procedure:

$$S^{WZ} = \frac{-iN_c}{240\pi^2} \int_\Omega d^5x \epsilon^{\mu\nu\lambda\rho\sigma} Tr(l_\mu l_\nu l_\lambda l_\rho l_\sigma), \quad (2)$$

where Ω is a 5-dimensional region with the 4-dimensional space-time as its boundary and $l_\mu = U^\dagger \partial_\mu U$. Action (2) defines important topological properties of skyrmions, but it does not contribute to the static masses of classical configurations [8, 9]. Variation of this action can be presented as a well defined contribution to the Lagrangian (integral over the 4-dimensional space-time).

We begin our calculations, however, with $U \in SU(2)$. The classical mass of $SU(2)$ solitons, in the most general case, depends on 3 profile functions: f, α and β and is given by

$$M_{cl} = \int \left\{ \frac{F_\pi^2}{8} [\vec{l}_1^2 + \vec{l}_2^2 + \vec{l}_3^2] + \frac{1}{2e^2} \left[[\vec{l}_1 \vec{l}_2]^2 + [\vec{l}_2 \vec{l}_3]^2 + [\vec{l}_3 \vec{l}_1]^2 \right] + \frac{1}{4} F_\pi^2 m_\pi^2 (1 - c_f) \right\} d^3r \quad (3)$$

Here $\vec{l}_k$ are the $SU(2)$ chiral derivatives defined by $U^\dagger \vec{\partial} U = i \vec{l}_k \tau_k$, where $k = 1, 2, 3$. The general parametrization of U_0 for an $SU(2)$ soliton we use here is given by $U_0 = c_f + s_f \vec{\tau}\vec{n}$

with $n_z = c_\alpha$, $n_x = s_\alpha c_\beta$, $n_y = s_\alpha s_\beta$, $s_f = sinf$, $c_f = cosf$, etc. For the rational map ansatz we are using here as starting configurations,

$$n_x = \frac{2Re\,R(\xi)}{1+|R(\xi)|^2}, \qquad n_y = \frac{2Im\,R(\xi)}{1+|R(\xi)|^2}, \qquad n_z = \frac{1-|R(\xi)|^2}{1+|R(\xi)|^2},$$

where $R(\xi)$ is a ratio of polynomials of the maximal power B of the variable $\xi = tg(\theta/2)exp(i\phi)$, θ and ϕ being polar and azimuthal angles defining the direction of the radius-vector $\vec{r}$. The explicit form of $R(\xi)$ is given in [2] for different values of B.

The "flavour" moment of inertia plays a very important role in the procedure of $SU(3)$ quantization [10]-[18], see formulas $(9), (10)$ below, and for arbitrary $SU(2)$ skyrmions is given by [17, 19]:

$$\Theta_F = \frac{1}{8}\int (1-c_f)\big[F_D^2 + \frac{1}{e^2}\big((\vec{\partial} f)^2 + s_f^2(\vec{\partial}\alpha)^2 + s_f^2 s_\alpha^2(\vec{\partial}\beta)^2\big)\big]d^3\vec{r}. \qquad (4a)$$

It is simply connected with $\Theta_F^{(0)}$ of the flavour symmetric case ($F_D = F_\pi$):

$$\Theta_F = \Theta_F^{(0)} + (F_D^2/F_\pi^2 - 1)\Gamma/4, \qquad (4b)$$

with Γ defined in (5) below. The isotopic momenta of inertia are the components of the corresponding tensor of inertia. They have been discussed in many papers, see e.g. [9]-[12], so, we will not present them here. For majority of multiskyrmions we discuss, this tensor of inertia is close to the unit matrix multiplied by the isotopic moment of inertia Θ_T. This is exactly the case for $B = 1$ and, to within a good accuracy, for $B = 3, 7$. Considerable deviations take place for the torus with $B = 2$, and smaller ones for $B = 4, 5, 6$ and 8, see **Table 1**. The quantity Γ (or Σ-term), which defines the contribution of the mass term to the classical mass of solitons, and $\tilde{\Gamma}$ are used directly in the quantization procedure. They are given by:

$$\Gamma = \frac{F_\pi^2}{2}\int (1-c_f)d^3\vec{r}, \qquad \tilde{\Gamma} = \frac{1}{4}\int c_f\big[(\vec{\partial} f)^2 + s_f^2(\vec{\partial}\alpha)^2 + s_f^2 s_\alpha^2(\vec{\partial}\beta)^2\big]d^3\vec{r}. \qquad (5)$$

The following relation can also be established: $\tilde{\Gamma} = 2(M_{cl}^{(2)}/F_\pi^2 - e^2\Theta_F^{Sk})$, where $M_{cl}^{(2)}$ is the second-order term contribution to the classical mass of the soliton, and Θ_F^{Sk} is the Skyrme term contribution to the flavour moment of inertia. The calculated masses of solitons, momenta of inertia Θ_F, Θ_T, Γ or Σ-term and $\tilde{\Gamma}$ are presented in **Table 1**.

B	M_{cl}	$\Theta_F^{(0)}$	Θ_T	$\Theta_{T,3}$	Γ	$\tilde{\Gamma}$	ω_s	ω_c	ω_b	c_s	c_c	c_b	$\bar{c}_s$	$\bar{c}_c$
1	1.702	2.04	5.55	5.55	4.83	15.6	0.309	1.542	4.82	0.28	0.27	0.52	0.54	0.91
2	3.26	4.18	11.5	7.38	9.35	22	0.293	1.511	4.76	0.27	0.24	0.49	0.53	0.90
3	4.80	6.34	14.4	14.4	14.0	27	0.289	1.504	4.75	0.40	0.37	0.58	0.60	0.92
4	6.20	8.27	16.8	20.3	18.0	31	0.283	1.493	4.74	0.47	0.44	0.62	0.64	0.92
5	7.78	10.8	23.5	19.5	23.8	35	0.287	1.505	4.75	0.42	0.40	0.60	0.62	0.92
6	9.24	13.1	25.4	27.7	29.0	38	0.287	1.504	4.75	0.48	0.46	0.63	0.67	0.93
7	10.6	14.7	28.9	28.9	32.3	44	0.282	1.497	4.75	0.48	0.46	0.64	0.66	0.93
8	12.2	17.4	33.4	31.4	38.9	47	0.288	1.510	4.79	0.49	0.47	0.64	0.67	0.93

Table 1. Characteristics of the bound states of skyrmions with baryon numbers up to $B = 8$. The classical mass of solitons M_{cl} is in Gev, momenta of inertia Θ_F, Θ_T and $\Theta_{T,3}$, Γ and $\tilde{\Gamma}$ - in Gev^{-1}, the excitation frequencies for flavour F, ω_F in GeV. $c_{s,c,b}$ and $\bar{c}_{s,c}$ are the hyperfine splitting constants for multibaryons defined in Eq.(21). The constant $\bar{c}_b$ is close to 0.99 for all B and is not included into the Table. The parameters of the model $F_\pi = 186\,MeV$, $e = 4.12$. The accuracy of calculations is better than 1% for the masses and few % for other static characteristics of solitons. The $B = 1$ quantities as well as $B = 2$

quantities for the torus calculated previously, are shown for comparison.

As can be seen from **Tables 1 and 2**, there are two "islands" of stability for baryon numbers considered here: at $B = 4$, which is not unexpected, and for $B = 7$, and this is something new and unexpected. So far, this property seems to be specific to the Skyrme model. The difference between Θ_T and $\Theta_{T,3}$ is maximal for the toroidal $B = 2$ configuration and decreases with increasing B. It vanishes for $B = 3$ and 7 precisely. The accuracy of calculations decreases with increasing baryon number. It is difficult to estimate it for such quantities as $\omega_{F,B}$ and $c_{F,B}$ where it depends also on the particular method of calculation - the rigid oscillator model in our case.

The behaviour of static properties of multiskyrmions and flavour excitation frequencies shown in **Table 1** is similar to that obtained in [22] for toroidal configurations with $B = 2, 3, 4$. We note that the flavour inertia $\Theta_{F,B}$ increases with B almost proportionally to B.

3 Flavour excitation frequencies and $\sim 1/N_c$ zero mode corrections

To quantize the solitons in their $SU(3)$ configuration space, in the spirit of the bound state approach to the description of strangeness proposed in [13]-[14] and used in [15]-[17], we consider the collective coordinate motion of the meson fields incorporated into the matrix U:

$$U(r,t) = R(t)U_0(O(t)\vec{r})R^\dagger(t), \qquad R(t) = A(t)S(t), \tag{6}$$

where U_0 is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the upper left hand corner), $A(t) \in SU(2)$ describes $SU(2)$ rotations and $S(t) \in SU(3)$ describes rotations in the "strange", "charm" or "bottom" directions and $O(t)$ describes rigid rotations in real space. For definiteness we consider the extension of the (u,d) $SU(2)$ Skyrme model in the (u,d,s) direction, when D is the field of K-mesons but it is clear that quite similar extensions can also be made in the directions of charm or bottom. So

$$S(t) = exp(i\mathcal{D}(t)), \qquad \mathcal{D}(t) = \sum_{a=4,...7} D_a(t)\lambda_a, \tag{7}$$

where λ_a are Gell-Mann matrices of the (u,d,s), (u,d,c) or (u,d,b) $SU(3)$ groups. The (u,d,c) and (u,d,b) $SU(3)$ groups are quite analogous to the (u,d,s) one. For the (u,d,c) group a simple redefiniton of hypercharge should be made. For the (u,d,s) group, $D_4 = (K^+ + K^-)/\sqrt{2}$, $D_5 = i(K^+ - K^-)/\sqrt{2}$, etc. For the (u,d,c) group $D_4 = (D^0 + \bar{D}^0)/\sqrt{2}$, etc.

The angular velocities of the isospin rotations $\vec{\omega}$ are defined in the standard way [9]: $A^\dagger \dot{A} = -i\vec{\omega}\vec{\tau}/2$. We shall not consider here the usual space rotations in detail because the corresponding momenta of inertia for BS are much greater than the isospin momenta of inertia, and for the lowest possible values of angular momentum J, the corresponding quantum correction is either exactly zero (for even B), or small, see also formulas (17) and (21) below.

The field D is small in magnitude. In fact, it is, at least, of order $1/\sqrt{N_c}$, where N_c is the number of colours in QCD, see Eq. (14). Therefore, the expansion of the matrix S in D can be made safely.

The mass term of the Lagrangian (1) can be calculated exactly, without expansion in the powers of the field D, because the matrix S is given by $S = 1 - i\mathcal{D}\ sind/d - \mathcal{D}^2(1-cosd)/d^2$

with $d^2 = Tr\mathcal{D}^2$. We find that

$$\Delta\mathcal{L}_M = -\frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{4}(1-c_f)s_d^2 \tag{8}$$

The expansion of this term can be done easily up to any order in d. The comparison of this expression with ΔL_M, within the collective coordinate approach of the quantization of $SU(2)$ solitons in the $SU(3)$ configuration space [10]-[12], allows us to establish the relation $sin^2 d = sin^2\nu$, where ν is the angle of the λ_4 rotation, or the rotation into the "strange" ("charm", "bottom") direction.

After some calculations we find that the Lagrangian of the model, to the lowest order in the field D, can be written as

$$L = -M_{cl,B} + 4\Theta_{F,B}\dot{D}^\dagger\dot{D} - \left[\Gamma_B\left(\frac{F_D^2}{F_\pi^2}m_D^2 - m_\pi^2\right) + \tilde{\Gamma}_B(F_D^2 - F_\pi^2)\right]D^\dagger D - i\frac{N_cB}{2}(D^\dagger\dot{D} - \dot{D}^\dagger D). \tag{9}$$

Here and below D is the doublet K^+, K^0 $(D^0, D^-$, or $B^+, B^0)$: $d^2 = Tr\mathcal{D}^2 = 2D^\dagger D$. We have kept the standard notation for the moment of inertia of the rotation into the "flavour" direction Θ_F for Θ_c, Θ_b or Θ_s [10]-[12]; different notations are used in [15, 16] (the index c denotes the charm quantum number, except in N_c). The contribution proportional to $\tilde{\Gamma}_B$ is suppressed in comparison with the term $\sim\Gamma$ by a small factor $\sim (F_D^2 - F_\pi^2)/m_D^2$, and is more important for strangeness.

The term proportional to N_cB in (9) arises from the Wess-Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavour and antiflavour in the general case) [13]-[16].

Following the canonical quantization procedure the Hamiltonian of the system, including the terms of the order of N_c^0, takes the form [15, 16]:

$$H_B = M_{cl,B} + \frac{1}{4\Theta_{F,B}}\Pi^\dagger\Pi + \left[\Gamma_B\bar{m}_D^2 + \tilde{\Gamma}_B(F_D^2 - F_\pi^2) + \frac{N_c^2B^2}{16\Theta_{F,B}}\right]D^\dagger D + i\frac{N_cB}{8\Theta_{F,B}}(D^\dagger\Pi - \Pi^\dagger D), \tag{10}$$

where $\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2$. The momentum Π is canonically conjugate to variable D (see Eq.(18) below). Eq. (10) describes an oscillator-type motion of the field D in the background formed by the (u,d) $SU(2)$ soliton. After the diagonalization, which can be done explicitly following [15, 16], the normal-ordered Hamiltonian can be written as

$$H_B = M_{cl,B} + \omega_{F,B}a^\dagger a + \bar{\omega}_{F,B}b^\dagger b + O(1/N_c) \tag{11}$$

with $a^\dagger$, $b^\dagger$ being the operators of creation of strangeness (i.e., antikaons) and antistrangeness (flavour and antiflavour) quantum number, $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ being the frequencies of flavour (antiflavour) excitations. D and Π are connected with a and b in the following way [15, 16]:

$$D^i = \frac{1}{\sqrt{N_cB\mu_{F,B}}}(b^i + a^{\dagger i}), \qquad \Pi^i = \frac{\sqrt{N_cB\mu_{F,B}}}{2i}(b^i - a^{\dagger i}) \tag{12}$$

with

$$\mu_{F,B} = [1 + 16(\bar{m}_D^2\Gamma_B + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B)\Theta_{F,B}/(N_cB)^2]^{1/2}. \tag{13}$$

For the lowest states the values of D are small:

$$D \sim \left[16\Gamma_B\Theta_{F,B}\bar{m}_D^2 + N_c^2B^2\right]^{-1/4}, \tag{14}$$

and increase, with increasing flavour number $|F|$, like $(2|F|+1)^{1/2}$. As was noted in [16], deviations of the field D from the vacuum decrease with increasing mass m_D, as well as with increasing number of colours N_c, and the method works for any m_D (and also for charm and bottom quantum numbers).

The excitation frequencies ω and $\bar{\omega}$ are:

$$\omega_{F,B} = \frac{N_c B}{8\Theta_{F,B}}(\mu_{F,B} - 1), \qquad \bar{\omega}_{F,B} = \frac{N_c B}{8\Theta_{F,B}}(\mu_{F,B} + 1). \tag{15}$$

As was observed in [17], the difference $\bar{\omega}_{F,B} - \omega_{F,B} = N_c B/(4\Theta_{F,B})$ coincides, to the leading order in N_c, with the expression obtained in the collective coordinate approach [18, 19]. At large m_D the $\mu_{F,B} \simeq 4\bar{m}_D(\Gamma_B \Theta_{F,B})^{1/2}/(N_c B)$ and for the difference $\omega_{F,1} - \omega_{F,B}$ we obtain ($N_c = 3$):

$$\omega_{F,1} - \omega_{F,B} \simeq \frac{\bar{m}}{2}\left[\left(\frac{\Gamma_1}{\Theta_{F,1}}\right)^{1/2} - \left(\frac{\Gamma_B}{\Theta_{F,B}}\right)^{1/2}\right] + \frac{3}{8}\left(\frac{B}{\Theta_{F,B}} - \frac{1}{\Theta_{F,1}}\right). \tag{16}$$

Obviously, at large m_D, the first term in (16) dominates and is positive if $\Gamma_1/\Theta_{F,1} \geq \Gamma_B/\Theta_{F,B}$. This is confirmed by looking at **Table 1**. Note also that the bracket in the first term in (16) does not depend on the parameters of the model if the background $SU(2)$ soliton is calculated in the chirally symmetrical limit: both Γ and Θ scale like $\sim 1/(F_\pi e^3)$. In a realistic case when the physical pion mass is included in (3) there is some weak dependence on the parameters of the model.

The FSB in the flavour decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.22$ and $F_D/F_\pi = 1.7 \pm 0.2$, should be taken into account. In the Skyrme model this fact leads to the increase of the flavour excitation frequencies which changes the spectra of flavoured $(c,\, b)$ baryons and puts them in a better agreement with the data [20, 21]. It also leads to some changes of the total binding energies of BS [17]. This is partly due to the large contribution of the Skyrme term to the flavour moment of inertia Θ_F. Note, that in [16] the FSB in strangeness decay constant was not taken into account, and this led to underestimation of the strangeness excitation energies. Heavy flavours (c, b) have not been considered in these papers.

The terms of the order of N_c^{-1} in the Hamiltonian, which depend on the angular velocities of rotations in the isospin and the usual space and which describe the zero-mode contributions, are not crucial but important for the numerical estimates of the spectra of baryonic systems. To calculate them one should first obtain the Lagrangian of BS including all the terms upto $O(1/N_c)$. The Lagrangian can be written in a compact form as:

$$L \simeq -M_{cl} + 4\Theta_{F,B}\big[\dot{D}^\dagger \dot{D}\big(1 - \frac{d^2}{3}\big) - \frac{2}{3}\big(D^\dagger \dot{D}\dot{D}^\dagger D - (D^\dagger \dot{D})^2 - (\dot{D}^\dagger D)^2\big)\big] + 2\Theta_{F,B}(\vec{\omega}\vec{\beta}) + \frac{\Theta_{T,B}}{2}(\vec{\omega} - \vec{\beta})^2 -$$

$$-[\Gamma_B \tilde{m}_D^2 + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B]D^\dagger D\big(1 - \frac{d^2}{3}\big) + i\frac{N_c B}{3}\big(1 - \frac{d^2}{3}\big)(\dot{D}^\dagger D - D^\dagger \dot{D}) - \frac{N_c B}{2}\vec{\omega} D^\dagger \vec{\tau} D, \tag{17a}$$

where $d^2 = 2D^\dagger D$ and

$$\vec{\beta} = -i(\dot{D}^\dagger \vec{\tau} D - D^\dagger \vec{\tau} \dot{D}).$$

For the axially symmetrical configurations, like the $B = 2$ torus, the term $\Theta_{T,B}(\omega_3 - \beta_3)^2/2$ in (17a) should be substituted by

$$\delta L = \frac{\Theta_{3,B}}{2}(\omega_3 - n\Omega_3 - \beta_3)^2 + \frac{\Theta_{J,B}}{2}(\Omega_1^2 + \Omega_2^2), \tag{17b}$$

where Ω_i are the components of the angular velocities of rotation in the usual space, $\dot{O}_{in} O_{kn} = \epsilon_{ikl}\Omega_l$. Taking into account the terms $\sim 1/N_c$ we find that the canonical variable Π conjugate to D is

$$\Pi = \frac{\partial L}{\partial \dot{D}^\dagger} = 4\Theta_{F,B}\big[\dot{D}\big(1 - \frac{d^2}{3}\big) - \frac{2}{3}D^\dagger \dot{D}\, D + \frac{4}{3}\dot{D}^\dagger D\, D\big] + i(\Theta_{T,B} - 2\Theta_{F,B})\vec{\omega}\vec{\tau} D - i\Theta_{T,B}\vec{\beta}\vec{\tau} D + i\frac{N_c B}{2}\big(1 - \frac{d^2}{3}\big)\, D. \tag{18}$$

¿From $(17a)$ the body-fixed isospin operator is:

$$\vec{I}^{bf} = \partial L/\partial\vec{\omega} = \Theta_{T,B}\vec{\omega} + (2\Theta_{F,B} - \Theta_{T,B})\vec{\beta} - \frac{N_c B}{2} D^\dagger \vec{\tau} D. \tag{19}$$

Using the identities :

$$-i\vec{\beta}\vec{\tau}D = 2D^\dagger D\dot{D} - (\dot{D}^\dagger D + D^\dagger \dot{D})D \tag{20a}$$

and

$$\vec{\beta}^2 = 4D^\dagger D\dot{D}^\dagger \dot{D} - (\dot{D}^\dagger D + D^\dagger \dot{D})^2 \tag{20b}$$

we find that the $\sim 1/N_c$ zero mode quantum corrections to the energies of skyrmions can be estimated [15, 16] as:

$$\Delta E_{1/N_c} = \frac{1}{2\Theta_{T,B}}\left[c_{F,B}T_r(T_r+1) + (1-c_{F,B})I(I+1) + (\bar{c}_{F,B} - c_{F,B})I_F(I_F+1)\right], \tag{21a}$$

where $I = I^{bf}$ is the value of the isospin of the baryon or BS, which can be written also as:

$$\vec{I}^{bf} = \Theta_T\vec{\omega} + \left(1 - \frac{\Theta_T}{2\Theta_F}\right)\vec{I}_F - \frac{N_c B\Theta_T}{4\Theta_F} D^\dagger\vec{\tau}D \tag{22}$$

with the operator $\hat{\vec{I}}_F = (b^\dagger\vec{\tau}b - a^T\vec{\tau}a^{\dagger T})/2$.

T_r is the quantity analogous to the "right" isospin T_r, in the collective coordinate approach [10, 18], and $\vec{T}_r = \vec{I}^{bf} - \vec{I}_F$. The hyperfine structure constants $c_{F,B}$ and $\bar{c}_{F,B}$ are defined by relations:

$$1 - c_{F,B} = \frac{\Theta_{T,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B} - 1), \qquad 1 - \bar{c}_{F,B} = \frac{\Theta_{T,B}}{\Theta_{F,B}(\mu_{F,B})^2}(\mu_{F,B} - 1). \tag{23}$$

To take into account the usual space rotations the J-dependent terms should be added to $(21a)$. For the axially-symmetrical configurations, like the $B = 2$ torus, they are equal to [18, 16]:

$$\Delta E_{1/N_c} = \left(\frac{1}{2n^2\Theta_{3,B}} - \frac{1}{2n^2\Theta_{T,B}} - \frac{1}{2\Theta_{J,B}}\right)(J_3^{bf})^2 + \frac{J(J+1)}{2\Theta_{J,B}}, \tag{21b}$$

with $\Theta_{J,B}$ being the moment of inertia corresponding to the usual space rotations - orbital moment of inertia, which is known to increase with increasing B-number almost proportionally to B^2 [17, 23]. For such configurations the body-fixed 3-d component of the angular momentum J_3^{bf} and the nonstrange part of the 3-d component of the isospin (also body-fixed) are connected by the relation $J_3^{bf} = -nT_{r,3}^{bf}$ (see, e.g. [18, 16] and references therein). Realistic cases of multiskyrmions are intermediate between the case of incoherence of usual space and isospace rotations and the complete coherence, as in $(21b)$ for the rotation relative to the axis of axial symmetry. However, the J-dependent terms of the type $(21b)$ are cancelled mostly in the differences of energies of states which belong to the same $SU(3)$ multiplet, *i.e.* when they have the same values of J, (p,q) and $T_{r,3}$.

In the case of antiflavour excitations we obtain the same formulas, with the substitution $\omega \to \bar{\omega}$ and $\mu \to -\mu$ in (23). For example,

$$\bar{c}_{\bar{F},B} = 1 + \frac{\Theta_{T,B}}{\Theta_{F,B}\mu_{F,B}^2}(\mu_{F,B} + 1). \tag{24}$$

The excitation energies for antiflavours are close to $\sim 0.59 Gev$ for antistrangeness, $\sim 1.75\, Gev$ for anticharm and to $\sim 4.95\, Gev$ for antibottom. However, these numbers should be considered as lower bounds only since to calculate them we have used a simplified version of the bound state soliton model.

4 Estimates of the spectra of multibaryons with strangeness, charm or bottom

In the bound state soliton model, and in its rigid oscillator version as well, the states predicted do not correspond originally to the definite $SU(3)$ or $SU(4)$ representations. How this can be remedied was shown in [16]; see also Eq. (26) – (29) below. The quantization condition $(p+2q)/3 = B$ [10], for arbitrary N_c, changes to $(p+2q) = N_cB + 3n_{q\bar{q}}$, where $n_{q\bar{q}}$ is the number of additional valent quark-antiquark pairs present in the quantized states [18]. For example, the state with $B = 1$, $|F| = 1$, $I = 0$ and $n_{q\bar{q}} = 0$ should belong to the octet of (u, d, s), or (u, d, c), $SU(3)$ group, if $N_c = 3$; see also [16]. The state with $B = 2$, $|F| = 2$ and $I = 0$ should belong to the 27-plet of the corresponding group, etc. The states having antiflavour quantum number, i.e. positive strangeness or bottom quantum number or negative charm should have the number of additional quark-antiquark pairs $n_{q\bar{q}} \geq |F|$ [18]. If $\Theta_F \to \infty$, Eqs. (21) go over into the expression obtained within the collective coordinate approach [10, 17]. In a realistic case, with $\Theta_T/\Theta_F^{(0)} \sim 2 - 2.7$, the structure of (21) is more complicated.

First we consider quantized states of BS which belong to the lowest possible $SU(3)$ irreps (p, q) for each value of the baryon number, $p+2q = 3B$: $p = 0$, $q = 3B/2$ for even B, and $p = 1$, $q = (3B-1)/2$ for odd B. For $B = 3$, 5 and 7 they are $\bar{35}$, $\bar{80}$ and $1\bar{43}$-plets, for $B = 2$, 4, 6 and 8 - $\bar{10}$, $\bar{28}$, $\bar{55}$ and $\bar{91}$-plets. Since we are interested in the lowest energy states, we discuss here the baryonic systems with the lowest allowed angular momentum, *ie* $J = 0$, for $B = 2$, 4, 6 and 8. For odd B the quantization of BS encounters some difficulties (see [23]), but the correction to the energy of quantized states due to the nonzero angular momentum is small and decreases with increasing B since the corresponding moment of inertia increases proportionally to $\sim B^2$ [22, 23]. Moreover, the J-dependent correction to the energy cancels in the differences of energies of flavoured and flavourless states.

For the energy difference between the state with flavour F belonging to the (p, q) irrep, and the ground state with $F = 0$ and the same angular momentum and (p, q) we obtain:

$$\Delta E_{B,F} = |F|\omega_{F,B} + \frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}}[I(I+1) - T_r(T_r+1)] + \frac{(\mu_{F,B}-1)(\mu_{F,B}-2)}{4\mu_{F,B}^2\Theta_{F,B}} I_F(I_F+1), \qquad (25)$$

where $T_r = p/2$ and usually $I_F = I - T_r$. Note that the moment of inertia Θ_T does not enter the difference of energies (25). Obviously, for "minimal" BS, i.e. those which do not contain additional quark-antiquark pairs,

$$T_r \leq 3B/2. \qquad (26)$$

The maximal isospin carried by $|F|$ flavoured mesons bound by (u, d) solitons satisfies another obvious relation:

$$I_F = |F|/2. \qquad (27)$$

Simple arguments allow us also to get the following restrictions on the total isospin of BS:

$$|T_r - |F|/2| \leq I \leq T_r + |F|/2 \qquad (28)$$

and

$$I \leq (3B - |F|)/2. \qquad (29)$$

The lowest of the two upper bounds should be taken as the final upper bound. It is easy to check that our bounds correspond to the known $SU(3)$ multiplets for each value of T_r.

For the $B=1$ case, the difference of masses within the octet of baryons, Λ_F and nucleon, Σ_F and Λ_F, is

$$\Delta M_{\Lambda_F N} = \omega_{F,1} - \frac{3(1-\bar{c}_{F,1})}{8\Theta_{T,1}} = \omega_{F,1} - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}}, \qquad \Delta M_{\Sigma_F\Lambda_F} = \frac{(1-c_{F,1})}{\Theta_{T,1}} = \frac{\mu_{F,1}-1}{2\mu_{F,1}\Theta_{F,1}}. \quad (30)$$

Clearly, there are cancellations in Eq. (25) - the binding energy differencies of multiskyrmions. For states with maximal isospin $I = T_r + |F|/2$ the energy difference can be simplified to:

$$\Delta E_{B,F} = |F|\left[\omega_{F,B} + T_r\frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} + \frac{(|F|+2)}{8\Theta_{F,B}}\frac{(\mu_{F,B}-1)^2}{\mu_{F,B}^2}\right]. \quad (31)$$

For even B $T_r = 0$, for odd B we should take $T_r = 1/2$ for the lowest $SU(3)$ irreps.

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$	$\Delta\epsilon_{s=-3}$	$\Delta\epsilon_{c=3}$	$\Delta\epsilon_{s=-4}$
2	-0.047	-0.027	0.02	-0.115	-0.088	0.02	-0.205	-0.183	-0.316
3	-0.042	-0.010	0.04	-0.098	-0.040	0.06	-0.168	-0.064	-0.252
4	-0.020	0.019	0.06	-0.051	0.022	0.10	-0.092	0.013	-0.144
5	-0.027	0.006	0.05	-0.063	0.001	0.08	-0.108	0.019	-0.160
6	-0.019	0.016	0.05	-0.045	0.023	0.10	-0.078	0.028	-0.117
7	-0.016	0.021	0.06	-0.041	0.033	0.11	-0.070	0.037	-0.105
8	-0.017	0.014	0.02	-0.040	0.021	0.03	-0.068	0.020	-0.100

Table 2. The binding energy differences $\Delta\epsilon_{s,c,b}$ are the changes of binding energies of lowest BS with flavour s, c or b and isospin $I = T_r + |F|/2$ in comparison with usual u, d nuclei, for the flavour numbers $S = -1, -2, -3$ and -4, $c = 1$, 2 and 3, $b = -1$ and -2 (see Eq. (32)).

It follows from (30) and (31) that when some nucleons are replaced by flavoured hyperons in BS the binding energy of the system changes by

$$\Delta\epsilon_{B,F} = |F|\left[\omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}} - T_r\frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} - \frac{(|F|+2)}{8\Theta_{F,B}}\frac{(\mu_{F,B}-1)^2}{\mu_{F,B}^2}\right]. \quad (32)$$

For strangeness Eq. (32) is negative indicating that stranglets should have binding energies smaller than those of nuclei, or can be unbound. Since $\Theta_{F,B}$ and $\Theta_{T,B}$ increase with increasing B and m_D this leads to the increase of binding with increasing B and mass of the flavoured state, in agreement with [17]. For charm and bottom Eq. (32) is positive for $B \geq 3$ or 4. It follows from **Table 2** that dibaryons with strangeness or charm quantum number are probably unbound, but those with $b = -1$ or $b = -2$ could be bound. The multibaryons with $B \geq 4$ and $S = -1$ can be bound, as well as multibaryons with $c = 1, 2$ or 3, or bottom $b = -1, -2$.

Had the momenta of inertia of BS at small values of B been proportional to the baryon number B, then the values of μ, excitation frequencies ω_F and coefficients c would not have depended on B at all. In this case the binding energy would have consisted only of its classical part and a contribution from zero modes; the difference of ω's would have been absent in this case.

The nuclear fragments with sufficiently large values of strangeness (or bottom) may have been found in experiments as fragments with negative charge Q, according to the well known relation, $Q = T_3 + (B+S)/2$ (similarly for the bottom number). Recently one event of a long lived nuclear fragment with mass about $7.4 Gev$ was reported in [24]. Using the above formulas it is not difficult to establish that this fragment may be the state with $B = -S = 6$, or $B = 7$ and strangeness $S = -4$, or -3, see also **Table 3** below. Greater

values of strangeness are not excluded since the method used here overestimates the flavour excitation energies, especially for smaller baryon numbers and for the strangeness quantum number.

Another case of interest involves considering the BS with isospin $I=0$. In this case $I_F = T_r = |F|/2$ and so such states do not belong to the lowest possible $SU(3)$ multiplet for each value of B (except for the case $|F|=1$). For the energy difference between this state and a flavourless state belonging to the same $SU(3)$ irrep it is easy to obtain:

$$\Delta E_{B,F} = |F|\left[\omega_{F,B} - \frac{(|F|+2)}{8\Theta_{F,B}}\frac{(\mu_{F,B}-1)}{\mu_{F,B}^2}\right]. \tag{33}$$

For the difference of binding energies of such a state and the ground (u,d) state with lowest values (p^{min}, q^{min}) we have the following estimate:

$$\Delta\epsilon_{B,F} = |F|\left[\omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\Theta_{F,1}\mu_{F,1}^2} + \frac{(|F|+2)}{8\Theta_{F,B}}\frac{(\mu_{F,B}-1)}{\mu_{F,B}^2}\right] - \frac{1}{2\Theta_{T,B}}[|F|(|F|+2)/4 - T_r^{min}(T_r^{min}+1)], \tag{34}$$

where $T_r^{min} = 0$, or $1/2$. Using this formula we find the values given in **Table 3.** For example, the $B=2$, $|F|=2$ state discussed previously in [18] and later in [16] belongs to the 27-plet of the corresponding $SU(3)$ group. In the case of strangeness it has appeared already, probably, as a virtual level in the $\Lambda\Lambda$ system [24].

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$	$\Delta\epsilon_{s=-3}$	$\Delta\epsilon_{c=3}$	$\Delta\epsilon_{b=-3}$
2	–	–	–	−0.075	−0.029	0.02	–	–	–
3	0.000	0.034	0.07	–	–	–	−0.083	0.002	0.09
4	–	–	–	−0.047	0.030	0.09	–	–	–
5	−0.003	0.032	0.06	–	–	–	−0.060	0.035	0.12
6	–	–	–	−0.044	0.025	0.09	–	–	–
7	0.000	0.040	0.07	–	–	–	−0.042	0.068	0.15
8	–	–	–	−0.039	0.023	0.03	–	–	–

Table 3. The binding energies differences of lowest flavoured BS with isospin $I=0$ and the ground state with the same value of B and $I=0$ or $I=1/2$, see Eq. (34). The first 3 columns are for $|F|=1$, the next 3 columns - for $|F|=2$, and the last 3 - for $|F|=3$. The state with the value of flavour $|F|$ belongs to the $SU(3)$ multiplet with $T_r = |F|/2$.

We can see from this **Table** that the $B=7,\ S=-3$ state has a binding energy smaller than the (u,d) nucleus by $42\,MeV$, i.e. it can still be stable with respect to the strong decay, if we take into account the uncertainty of our estimates (recall that the nucleus ^7Li has the total binding energy $39\,Mev$.) The state with isospin equal to 2 - maximal value for $S=-3$ within the $(1,10)$ $SU(3)$ multiplet - has somewhat greater binding energy, see **Table 2** . The difference of energies of states with isospin $I = I^{max} = T_r + |F|/2$ and $I=0$ and the same value of F can be written as

$$E^{I^{max}} - E^{I=0} = \Delta\epsilon_{B,F}^{I=0} - \Delta\epsilon_{B,F}^{I^{max}} = \frac{|F|(|F|+2)}{8}\left(\frac{\mu_{F,B}-1}{\Theta_{F,B}\,\mu_{F,B}} - \frac{1}{\Theta_{T,B}}\right) - \frac{T_r(T_r+1)}{2\Theta_{T,B}} + T_r|F|\frac{\mu_{F,B}-1}{4\Theta_{F,B}\,\mu_{F,B}}\Bigg] \tag{35a}$$

At large $|F|$ this is approximately given by

$$E^{I^{max}} - E^{I=0} \simeq \frac{\vec{I}_F^2}{2\Theta_{T,B}}\left(1\ -\ 2c_{F,B}\right) \tag{35b}$$

At large B and F the isoscalar states have smaller energy if $c_{F,B} \leq 0.5$, see also **Table 1**.

It is of interest to consider the case corresponding to the bulk of "flavoured matter", i.e. $p=q=B=|F|$. Such "multilambda" states with isospin equal to zero have the

following value of $\Delta\epsilon$ for $B \gg 1$:

$$\Delta\epsilon \simeq |F|\left[\omega_{F,1} - \omega_{F,B} + \frac{|F|+2}{8}\left(\frac{\mu_{F,B}-1}{\Theta_{F,B}\mu_{F,B}^2} - \frac{1}{\Theta_{T,B}}\right) - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}}\right]. \quad (36)$$

At large $|F|$ the sign of this expression depends on the sign of the difference $(\mu_{F,B} - 1)/(\Theta_{F,B}\mu_{F,B}^2) - 1/\Theta_{T,B}$. To draw the final conclusion we need the knowledge of the behaviour of the ratio $\Theta_{T,B}/\Theta_{F,B}$ at large B. For heavy flavours, c and b, $\mu_{F,B} \gg 1$ and second term in (35) is negative, unless $\Theta_{T,B} \sim \mu_{F,B}\Theta_{F,B}$ which is not realistic (we have usually $\mu_s \sim 3$, $\mu_c \sim 15$ and $\mu_b \sim 73$). So, for heavy flavours it is not possible to obtain, in this way, the bulk of flavoured matter as quantized coherent multiskyrmions. Other possibilities remain to be investigated, e.g. flavoured skyrmion crystals.

As in the $B = 1$ case [26], the absolute values of masses of multiskyrmions are controlled by the poorly known loop corrections to the classic mass, or the Casimir energy. And as has been done for the $B = 2$ states, [18], the renormalization procedure is necessary to obtain physically reasonable values of these masses. As the binding energy of the deuteron is $30\ MeV$ instead of the measured value $2.2\ MeV$ we see that $\sim 30\ MeV$ characterises the uncertainty of our approach [17, 18]. But this uncertainty cancels in the differences of binding energies calculated in **Tables 2,3**.

5 Conclusions

Using rational map ansaetze as starting configurations we have calculated the static characteristics of bound skyrmions with baryon numbers up to 8. The excitation frequencies for different flavours - strangeness, charm and bottom - have been estimated using a rigid oscillator version of the bound state approach of the chiral soliton models. One notes that, in comparison with strangeness, this approach works even better for c and b flavours [20, 21]. Our previous conclusion that BS with charm and bottom have more chance to be bound by strong interactions than strange BS [17] is reinforced by the present investigation. Estimates of the binding energy differences of flavoured and flavourless states have some uncertainty, about few tens of Mev, but the tendency for charm and bottom to be bound stronger than strangeness is very clear.

A natural question now arises as to how these results depend on the choice of the parameters of the model. A set of parameters, which has been used extensively in the literature, is, e.g. the set introduced in [9] where the masses of the nucleon and the Δ isobar have been fitted in the massless case and with physical pion mass, correspondingly. In view of the large negative contribution of the loop corrections, or the Casimir energy, we feel that this choice of parameters cannot be taken too seriously. But calculations show that our results hold for this choice too. The energies of the flavour excitation are somewhat smaller, however: for example, for strangeness $\omega_s = 255\,MeV$ for $B = 1$ and $249\,MeV$ for $B = 4$, and we take $F_K/F_\pi = 1.22$ ($F_\pi = 108\,Mev$, $e = 4.84$). Similar changes take place for charm and bottom, and the conclusion that charmed or bottomed BS have good chances to be stable against strong interactions remains valid [17].

It should be kept in mind that corrections of the order of $1/N_c^2$ can lead to some change of our results. For example, the flavour moment of inertia changes to [18]

$$\Theta_F \to \Theta_F^{(0)} - D^\dagger D \frac{F_D^2 - F_\pi^2}{8} \int (1-c_f)(2-c_f)d^3r, \quad (37)$$

Decrease of the moment of inertia could lead to some increase of the zero-mode quantum corrections.

The apparent drawback of our approach is that the motion of the system into the "strange", "charm" or "bottom" directions is considered independently from other motions. Consideration of the BS with "mixed" flavours is possible in principle, but its treatment would be more involved (see, e.g. [27] where the collective coordinate approach to the quantization of $SU(n)$ skyrmions has been investigated).

Our results agree qualitatively with the results of [28] where the strangeness excitation frequencies had been calculated within the bound state approach. The difference is, however, in the behaviour of excitation frequencies: we have found that they decrease when the baryon number increases from $B = 1$ to 4, thus increasing the binding energy of the corresponding BS. This behaviour seems to be quite natural: there is an attraction between K, D or B meson field and $B = 1$ nucleon, and the attraction of a meson by $2, 3$ etc. nucleons is greater. Similar results hold for ordinary nuclei: the binding energy of a deuteron is $2.22\,Mev$ only, for $B = 3$ it is about $8\,Mev$, for $B = 4$ it is already $28\,Mev$, and saturation takes place soon.

There is a further difference between the rigid oscillator variant of the bound state model we have used here and the collective coordinate approach of soliton models studied previously [10]-[12]. In the collective coordinate approach involving zero modes of solitons with a rigid or a soft rotator variant of the model, the masses of baryons are usually considerably greater than in the bound state approach when the Casimir energies are not taken into account [26, 29]. One of the sources of this difference is the presence of a term of order N_c/Θ_F in the zero-mode contribution to the rotation energy, which is absent in the bound state model. Recently it has been shown by Walliser, for the $B = 1$ sector within the $SU(3)$ symmetrical $(m_K = m_\pi)$ variant of the Skyrme model [29], that this large contribution is cancelled almost completely by the kaonic 1-loop correction to the zero-point Casimir energy which is of the same order of magnitude, N_c^0 [29]. This correction has also been recently calculated within the bound state approach to the Skyrme model [30].

The charmed baryonic systems with $B = 3, 4$ were considered in [31] within a potential approach. The $B = 3$ systems were found to be very near the threshold and the $B = 4$ system was found to be stable with respect to the strong decay, with a binding energy of $\sim 10\,MeV$.

Experimental searches for the baryonic systems with flavour different from u and d could shed more light on the dynamics of heavy flavours in few-baryon systems. The negative charge fragment seen in the $NA52$ CERN experiment [25] could be explained in our approach as a quantized $B = 7$ skyrmion with strangeness $S = -3$ or -4. The other possibility is $B = 6$ and $S = -6$ or -7. The value of strangeness can be greater since the rigid oscillator version of the model we consider here overestimates the strangeness excitation energies.

The threshold for the charm production on a free nucleon is about $12GeV$, and for the double charm it is $\sim 25.2\,GeV$. For bottom, the threshold on a nucleon is ~ 70 GeV. However, for nuclei as targets the thresholds are much lower due to the two-step processes with mesons in intermediate states and due to the normal Fermi-motion of nucleons inside the target nucleus (see, e.g. [32]). Therefore, the production of baryons or baryonic systems with charm and bottom should be feasable in proton accelerators with energies of several tens of GeV, as well as in heavy ions collisions.

Let us finish by adding that a shortened and less complete version of this paper is available [33]. The results obtained recently in [34] within the detailed version of the bound state approach are in fair agreement with our, but the binding obtained in [34] is smaller than what we have found here. It should be noted that, in difference from [34], we have used the empirical values of flavour decay constants, taken into account the $1/N_c$ zero modes contributions to the energy of multibaryons and have considered only

the difference of binding energies of flavoured BS and the ground states where many of uncertainties cancel out.

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